Learning oligopolistic competition in electricity auctions

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Electricity Markets

• Worldwide deregulation in the 90’s: from monopolistic state-owned suppliers to “competitive” electricity market

• Active European Power Exchanges:
  – North Pool (Scandinavian), 1996
  – OMEL (Spain), 1998
  – APX (The Netherlands), 1999
  – NETA (UK), 2000
  – EEX Frankfurt (Germany), 2000
  – LPX Lipsia (Germany), 2000
  – PPE (Poland), 2000
  – Opcom (Romania), 2001
  – Powernext (France), 2001
  – Borzen (Slovenia), 2002
  – EXAA (Austria), 2003
  – IPEX (Italy), 2004
Electricity Markets

• Day-Ahead Market – DAM (Energy market)
  – Collection of offers and bids for next day hours
  – Construction of demand and supply curves
  – Market clearing for every hour of the next day
  – Zonal splitting, in the case of congestion

• Adjustment Market – AM (Energy market)
  – Allows revision of trading activities
  – Starts after DAM and considers separately every hour of the next day

• Ancillary Services Market – ASM (Service market)
  – Procures resources for dispatching, i.e. management, operation and control of the power system
  – Planned grid congestion relief, purchase of operating reserve for the next day, electricity for real-time balancing of the system
Agent-based modeling of Power Exchange

- Main aims and opportunities:
- To understand and to simulate the micro-structure of a real power exchange
- To reproduce the main stylized facts of the electricity price time-series
- To perform What-if Analysis
- To develop a framework for market design and validation
Computational setting

- No capacity constraints
  - Fully connected graph
- Inelastic and constant demand by means of a representative buyer
- Duopolistic competition, modeled by means of learning capabilities
  - Linear cost functions with different marginal costs

- Sellers’ strategy space is discrete and bi-dimensional
  - Couples of values for prices and quantities
- Two key questions are issued
  - What is the strategy most frequently chosen and how seller profits evolve in time?
Why behavioral, why learning?

• Real electricity markets are characterized by:
  – Small number of suppliers
    • possible oligopolistic scenario
  – Few big producers may exercise market power
  – Repeated interaction among the same sellers may produce collusive behavior
Reinforcement learning

Key features

• No Belief-Based
  – No exact behavioral rules are provided with the algorithm

• Minimal information for player knowledge
  – Own past actions and realized payoffs

• Explore and exploit stochastic search mechanism
  – Strategies that were successful in the past more likely will be employed in the future

• Structure of the game not needed

• Generic algorithmic structure:
  – Mixed strategies update: $\sigma^i_t = f(\sigma^i_{t-1}, \pi^i_{t-1})$
  – The selection of action at time $(t+1)$ is performed through a random draw according to mixed strategy distribution
**Strategic form of the game**

**R-bimatrix game (one-shot game)**

\[ a_j \in A_1, \ b_k \in A_2 \] actions of the matrix game

\[ R^i_{lm} \] istantaneous reward for player \( i \)

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Strategic form of the game

Q-bimatrix game (repeated games)

\( \alpha_j = \{a_{j1}, a_{j2}, ..., a_{jt}, ...\} \), \( \beta_k = \{b_{k1}, b_{k2}, ..., b_{kt}, ...\} \) stationary policies

\( Q_{im}^i \) expected sum of discounted delayed rewards for player \( i \)

\[
Q_{im}^i = E^\pi \left[ \sum_{t=0}^{\infty} \gamma^t R_t^i \right]
\]

\( \pi \) is a joint policy(\( \alpha_l, \beta_m \))

\[
\begin{array}{ccc}
\alpha_1 & \beta_1 & \beta_2 \\
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Q_{11}^1, Q_{11}^2 & Q_{12}^1, Q_{12}^2 \\
Q_{21}^1, Q_{21}^2 & Q_{22}^1, Q_{22}^2 \\
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Learning Algorithms

• Adaptive evolutionary - Marimon and McGrattan (1995)
  – Matrix game payoffs
  – Learning instantaneous rewards
  – Adaptive Stochastic algorithms

• Q-Learning – Watkins (1989)
  – Repeated game payoffs
  – Learning from delayed rewards
  – Sequential decision task
  – Markov Decision Process Framework
Solution Concepts

Joint set of actions \( x^* = (a_i^*, a_{-i}^*), \quad a_i^* \in A_i \)

\( \Pi_i \) is the generic payoff either \( R_i \) or \( Q_i \)

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Nash equilibria: competitive Solution

\( x^* = (a_i^*, a_{-i}^*) \) is Nash if \( \Pi_i(a_i^*, a_{-i}^*) \geq \Pi_i(a_i, a_{-i}^*), \quad \forall i \)

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Pareto optima: Tacit collusive solution

\( x^* \) is not Pareto if \( \exists x : \Pi_i(x) \geq \Pi_i(x^*), \quad \forall i \)
Two mechanism for market clearing

Uniform auction
System marginal price

Discriminatory auction
Pay as bid mechanism
Low-Demand Economic Scenario

Low Demand (LD) characteristics
A single seller can satisfy the whole demand

\[ Q^d < \min \{ Q_1^*, Q_2^* \} \quad Q_i^* = Q_2^* \quad \text{costs}_1 < \text{costs}_2 \]

Computational experiments

1. LD-MM: Low-demand computational experiment with MM adaptive evolutionary learning
2. LD-Q: Low-demand computational experiment with Q-learning

Frequencies are evaluated as ensemble averages over 10,000 experiments.
LD-MM case
(Nash & Pareto)

Uniform Auction case

Discriminatory Auction case
LD-MM case
(Profits)

Uniform Auction case

Discriminatory Auction case
Remarks to LD-MM

SOLUTIONS:
• In the uniform auction case, Nash equilibria are also Pareto optima
• In the long-run Nash solutions prevail in the context of a matrix game, as expected

AUCTION MECHANISM:
• Uniform auction mechanism is less efficient than the discriminatory auction mechanism
Low-Demand Economic Scenario

**Low Demand (LD) characteristics**
A single seller can satisfy the whole demand

\[
Q^d < \min \{Q_1^s, Q_2^s\} \quad Q_1^s = Q_2^s \quad \text{costs}_1 < \text{costs}_2
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Computational experiments

1. **LD-MM**: Low-demand computational experiment with MM adaptive evolutionary learning
2. **LD-Q**: Low-demand computational experiment with Q-learning

Frequencies are evaluated as ensemble averages over 10,000 experiments.
LD-Q (Nash & Pareto)

### Uniform Auction case

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### Discriminatory Auction case

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LD-Q case (Profits)

Uniform Auction case

Discriminatory Auction case
Remarks to LD-Q

SOLUTIONS:
• In the long-run Pareto solutions of the matrix game prevail. Folk theorem suggests this.

PROFITS:
• Producers realize similar profits

AUCTION MECHANISM:
• Within the LD-Q collusive behaviors arise
  – The demand side of the market is penalized.
Antitrust Action

• Agent-based artificial power exchange allows evaluation of the effectives of policies and actions through what-if analysis

• Market condition originated by splitting the overall productive capacity over three sellers

• Low Demand characteristics

\[
\max \{Q_1^s, Q_2^s, Q_3^s\} < Q^d \leq \min \{Q_i^s + Q_j^s\}
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Antitrust Action case
Nash & Pareto

Uniform Auction case

Discriminatory Auction case
Antitrust Action case

Profits

Uniform Auction case

Discriminatory Auction case
Remarks

• The Nash equilibrium strategies are no longer Pareto optima both in the Uniform and in the Discriminatory Auctions
• At the beginning, the frequency of Pareto optima grows at a faster rate than those of Nash equilibria
• In the long run, competition prevails
• Profits realized by sellers are lower than in the low-demand case with two sellers
  – Redistribution of productive capacity to a larger number of sellers may result an effective antitrust policy
Conclusions

RESULTS

• Behavioural modelling points out the role of repeated interactions among agents in power exchanges
• Discriminatory auction results more efficient than uniform auction

TAKE HOME MESSAGE
Agent-based technologies provide a fruitful approach to address complex problems of market design and policy analysis
Acknowledgements

• GAPEX is under development within financial support of the University of Genoa, the Italian Ministry of University (FIRB 2001 and COFIN 2004), EGL Italia S.p.A. an by the European Union under NEST PATHFINDER STREP Project COMPLEXMARKETS

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Adaptive evolutionary learning

(Marimon and McGrattan ‘95)

**Strength vector**

\[ S_{i,t}(a_i) = \begin{cases} 
S_{i,t-1}(a_i) - \frac{1}{\eta_{i,t-1}(a_i)} \left[ S_{i,t-1}(a_i) - \Pi_{i,t-1}(a_i) \right] & \text{if } i \text{ plays } a_i \\
S_{i,t-1}(a_i) & \text{otherwise}
\end{cases} \]

**Counter**

\[ \eta_{i,t}(a_i) = \begin{cases} 
\eta_{i,t-1}(a_i) + 1 & \text{if } i \text{ plays } a_i \\
\eta_{i,t-1}(a_i) & \text{otherwise}
\end{cases} \]

**INERTIA**

Updating mixed strategies

\[ \sigma_{i,t}(a_i) = \begin{cases} 
\sigma_{i,t-1}(a_i) + \frac{\eta_{i,t-1}(a_i) - \sum_{a_k} \sigma_{i,t-1}(a_k) e^{\delta_{i,t-1}(a_k)}}{\sum_{a_k} \sigma_{i,t-1}(a_k)} & \text{if } i \text{ plays } a_i \\
\sigma_{i,t-1}(a_i) & \text{otherwise with probability } \rho_{i,t} \text{ with probability } 1 - \rho_{i,t}
\end{cases} \]

**EXPERIMENTATION**

Minimum probability bound over pure strategies

\[ \sigma_{i,t}(a_i) = \begin{cases} 
\varepsilon_{i,t} & \text{if } \sigma_{i,t}(a_i) \leq \varepsilon_{i,t} \\
\frac{\overline{\sigma}_{i,t}(a_i)}{\sum_{a_i} \overline{\sigma}_{i,t}(a_i)} \left( 1 - \varepsilon_{i,t} \right) \cdot \left| \overline{\sigma}_{i,t}(a_i) \leq \varepsilon_{i,t} \right| & \text{otherwise}
\end{cases} \]

\( \varepsilon_{i,t} \) is the minimum probability value for mixed strategies.
Q-learning (Watkins ‘89)

Optimal Q-function

\[ Q^* (s, a) = R(s, a) + \gamma \sum_{s'} P(s, a, s') V^* (s) \]

Optimal value function

\[ V^* (s) = \max_a Q^* (s, a) \]

Stochastic Iterative algorithm

\[ Q_{t+1} (s_t, a_t) = (1 - \eta_t) Q_t (s_t, a_t) + \eta_t \left( R(s_t, a_t) + \gamma \max_a Q_t (s_t, a_t) \right) \]